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Three grade Manpower models having two sources of Depletion with Correlated Loss of Manpower and an Extended Threshold

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Abstract- A three grade marketing organization in which attrition lead to random loss of manpower due to policy decisions and transfer decisions is considered. This attrition of personnel will adversely affect the smooth functioning of the organization. Frequent recruitment is not advisable as it involves more cost. Since the loss of manpower and the inter-decision times are probabilistic, the organization requires a suitable recruitment policy to plan for recruitment. In this paper, the problem of time to recruitment based on shock model approach is studied by considering three different models of inter-policy decision times with correlated loss of manpower and the extended threshold which gives a better allowable cumulative loss of manpower in the organization using a univariate CUM policy of recruitment. Analytical expressions for the performance measures namely mean and variance for the time to recruitment are obtained for the loss of manpower in the organization. The results are numerically illustrated by assuming specific distributions and relevant conclusions are made.

Index Terms- Three grade manpower system, Correlated loss of manpower, Inter-policy decision times, Inter-transfer decision times, an Extended threshold, Univariate CUM policy of recruitment, Shock model.

1. INTRODUCTION

Exit of personnel is a common phenomenon in any marketing organization. This leads to reduction in the total strength of marketing personnel and will adversely affect the sales turnover of the organization, if recruitment is not planned. Frequent recruitments may also be expensive due to the cost of recruitments and training. As the loss of manpower is unpredictable, suitable recruitment policy has to be designed to overcome this loss. Several researchers have studied the problem of time to recruitment for a two grade manpower system using shock model approach. In this context, the authors in [1], [2] and [3] have given the stochastic models for manpower planning and social processes for the usefulness to construct the manpower models. The authors in [7] have obtained the mean and variance of the time to recruitment for an organization consisting of two grades (two grade manpower system) by assuming that the distribution of loss of man manpower in different decisions and that of the inter-decision times as exponential according as the threshold for the loss of man power in the organization is maximum (minimum) of the exponential thresholds in the two grades. Recently, in [4] and [5], the authors have obtained the stochastic model for time to recruitment under two sources of depletion of manpower using univariate policy of recruitment by considering various assumptions for breakdown thresholds, loss of manpower and the inter-policy decisions. This paper analyses the research work by assuming that the loss of manpower due to policy decisions forms a sequence of exchangeable and constantly correlated exponential random variables, the stochastic models are constructed and the inter-policy decisions times are in the following cases: (Model-I) as a sequence of independent and identically distributed hyper-exponential random variables, (Model-II) as a sequence of exchangeable and constantly correlated exponential random variables and (Model-III) as a geometric process. An attempt has been made to study the problem of time to recruitment for a three grade manpower system with an extended threshold and correlated loss of manpower. An extended threshold is introduced to give a better allowable cumulative loss of manpower in the manpower system. It is assumed that the inter-policy decisions times for the three grades form the same ordinary renewal process, the inter-transfer decisions times for the three grades form the same ordinary renewal process which is different from that of inter-policy decisions. The conventional breakdown threshold used in all the earlier studies is identified as the level of alertness in the present paper. If the organization is not alert when the cumulative loss of manpower exceeds this level of alertness, an allowable loss of manpower of magnitude D is permitted. However, recruitment has to be done when the cumulative loss of manpower exceeds this extended threshold. A univariate recruitment policy, usually known as CUM policy of recruitment in the literature, is based on the replacement policy associated with the shock model approach in reliability theory and is stated as follows: ***Recruitment is made whenever the cumulative loss of manpower exceeds the extended threshold***. Analytical results related to time to recruitment are derived by using univariate CUM policy of recruitment for all the three models. The influence of the nodal parameters on the performance measures are studied and relevant conclusions are made with the help of numerical illustrations.

2. MODEL DESCRIPTION

Consider an organization with three grades taking policy and transfer decisions at random epochs in (0,∞). Let $X_{ni}, n = A, B, C \;\&\; i = 1,2,3, \ldots$ be a sequence of random variables representing the loss of manpower in grade A,B,C due to ith policy decision which forms a sequence of exchangeable and constantly correlated exponential random variables with mean $\alpha_{1n}, n = A, B, C$ and correlation R_1, R_2, $R_3 \in [-1,1]$ with relation $b_m = \alpha_{1n}(1-R_m)$, m=1,2,3 & n=A,B,C. Let $\widetilde{X_m}$ be the cumulative loss of manpower for the three grades in the first m policy decisions. Let Y_{Aj}, Y_{Bj} and Y_{Cj} be independent and identically distributed exponential random variables representing the loss of manpower in the organization due to jth transfer decision with mean $1/\alpha_{2A}, 1/\alpha_{2B}, 1/\alpha_{2C}, \alpha_{2A}, \alpha_{2B}, \alpha_{2C} > 0$. Let $\widetilde{Y_n}$ be the cumulative loss of manpower for the three grades in the first n transfer. For i=1,2,…, let U_i be **(Model-I)** a sequence of independent and identically distributed hyper-exponential random variables representing the time between (i-1)th and ith policy decisions with mean $(p_1/\lambda_1) + ((1-p_1)/\lambda_2)$, $0 < p_1 < 1$, where p_1 is the proportion of policy decisions having high attrition rate $\lambda_1 > 0$ and $(1-p_1)$ is the proportion of policy decisions having low attrition rate $\lambda_2 > 0$, **(Model-II)** a sequence of exchangeable and constantly correlated exponential random variables with mean λ and correlation R∈[-1,1] with relation b=λ(1-R) and **(Model-III)** a geometric process with independent and identically distributed hyper-exponential distribution representing the time between (i-1)th and ith policy decisions with mean $(p_1/\lambda_1) + ((1-p_1)/\lambda_2)$, $0 < p_1 < 1$, where p_1 is the proportion of policy decisions having high attrition rate $\lambda_1 > 0$ and $(1-p_1)$ is the proportion of policy decisions having low attrition rate $\lambda_2 > 0$. For j=1,2,…, let V_j be independent and identically distributed hyper-exponential random variable representing the time between (j-1)th and jth transfer decisions with mean $(p_2/\lambda_3) + ((1-p_2)/\lambda_4)$, $0 < p_2 < 1$, where p_2 is the proportion of transfer decisions having high attrition rate $\lambda_3 > 0$ and $(1-p_2)$ is the proportion of transfer decisions having low attrition rate $\lambda_4 > 0$. Let $N_P(t)$ and $N_{Tr}(t)$ be the number of policy and transfer decisions taken in the organization during the period of recruitment (0,t]. Let $\widetilde{X_{N_P(t)}}$ and $\widetilde{Y_{N_{Tr}(t)}}$ be the total loss of manpower in $N_P(t)$ decisions and $N_{Tr}(t)$ decisions. Let the cumulative distribution function of the random variable K be $W_K(.)$ (density function $w_K(.)$), and the Laplace transform of $w_K(.)$be $\overline{w}_K(.)$. Assume that Z_A, Z_B and Z_C represents the threshold levels for the cumulative loss of manpower in grade A, B and C with mean $1/\theta_A, 1/\theta_B, 1/\theta_C$, respectively, where $\theta_A, \theta_B, \theta_C > 0$. Let Z be the threshold level for the cumulative loss of manpower for the entire organization and D represents the extended threshold with mean $\frac{1}{\theta_D}$ respectively, where $\theta_D > 0$. Let T be the time to recruitment for the entire organization. Here, X_i and Y_j are linear and cumulative and Z, X_j, Y_j are statistically independent.

2.1. *Analytical Results*

The event of time to recruitment is defined as follows: Recruitment occurs beyond t (t > 0) if and only if the total loss in manpower upto $N_P(t)$ policy decisions and $N_{Tr}(t)$ transfer decisions does not exceeds the breakdown threshold of the organization and it is given by

$$\{T > t\} \Leftrightarrow \{\widetilde{X}_{N_P(t)} + \widetilde{Y}_{N_{Tr}(t)} < Z\} \quad (1)$$

Hence the probability of occurences of these two events are equal.

$$P(T > t) = P[\widetilde{X}_{N_P(t)} + \widetilde{Y}_{N_{Tr}(t)} < Z] \quad (2)$$

Invoking the law of total probability and the result of renewal theory, the survival function of time to recruitment is determined. The rth moment for the time to recruitment is determined by taking the rth derivative of the Laplace transform of density function for the random variable with respect to s, and at s=0. Using this result the fundamental performance measures like mean and variance of time to recruitment is determined. Let Z = $(Z_A+Z_B+Z_C)+D$. Conditioning upon D, we get the distribution of the threshold and taking derivative for the Laplace transform of T at s=0, gives the mean time to recruitment.

$$E(T) = (C_1 - C_2 + C_3)T_D - C_1T_1 + C_2T_2 - C_3T_3 \quad (3)$$

Model-I:

In Model-I, Inter-policy decision times are assumed to form a sequence of independent and identically distributed hyper exponential random variables. In Eq.(3), we substitute,

$$T_D=\left[\begin{array}{l}\left(\dfrac{\psi_1\psi_2-(1-F)f_1}{\psi_1\psi_2}\right)\displaystyle\sum_{m=0}^{\infty}\left[\begin{array}{l}m\overline{w}_U^{\,m-1}(0)\overline{w}_U{}'(0)\\-(m+1)\overline{w}_U^{\,m}(0)\overline{w}_U{}'(0)\end{array}\right]E\\-\left(\dfrac{(1-F)(\psi_1f_2-f_1)}{\psi_2-\psi_1}\right)\displaystyle\sum_{m=0}^{\infty}\dfrac{\overline{w}_U^{\,m}(\psi_1)}{\psi_1^{\,2}}E\\-\left(\dfrac{(1-F)(f_1-f_2\psi_2)}{\psi_2-\psi_1}\right)\displaystyle\sum_{m=0}^{\infty}\dfrac{\overline{w}_U^{\,m}(\psi_2)}{\psi_2^{\,2}}E\\+\left(\dfrac{(1-F)(\psi_1f_2-f_1)}{\psi_2-\psi_1}\right)\displaystyle\sum_{m=0}^{\infty}\dfrac{\overline{w}_U^{\,m+1}(\psi_1)}{\psi_1^{\,2}}E\\+\left(\dfrac{(1-F)(f_1-f_2\psi_2)}{\psi_2-\psi_1}\right)\displaystyle\sum_{m=0}^{\infty}\dfrac{\overline{w}_U^{\,m+1}(\psi_2)}{\psi_2^{\,2}}E\end{array}\right]$$

For i=1,2,3 and j=1,3,5

$$T_i=\left[\begin{array}{l}\left(\dfrac{\gamma_j\gamma_{j+1}-(1-B_i)d_j}{\gamma_j\gamma_{j+1}}\right)\displaystyle\sum_{m=0}^{\infty}\left[\begin{array}{l}m\overline{w}_U^{\,m-1}(0)\overline{w}_U{}'(0)\\-(m+1)\overline{w}_U^{\,m}(0)\overline{w}_U{}'(0)\end{array}\right]A_i\\-\left(\dfrac{(1-B_i)(\gamma_jd_{j+1}-d_j)}{\gamma_{j+1}-\gamma_j}\right)\displaystyle\sum_{m=0}^{\infty}\dfrac{\overline{w}_U^{\,m}(\gamma_j)}{\gamma_j^{\,2}}A_i\\-\left(\dfrac{(1-B_i)(d_j-d_{j+1}\gamma_{j+1})}{\gamma_{j+1}-\gamma_j}\right)\displaystyle\sum_{m=0}^{\infty}\dfrac{\overline{w}_U^{\,m}(\gamma_{j+1})}{\gamma_{j+1}^{\,2}}A_i\\+\left(\dfrac{(1-B_i)(\gamma_jd_{j+1}-d_j)}{\gamma_{j+1}-\gamma_j}\right)\displaystyle\sum_{m=0}^{\infty}\dfrac{\overline{w}_U^{\,m+1}(\gamma_j)}{\gamma_j^{\,2}}A_i\\+\left(\dfrac{(1-B_i)(d_j-d_{j+1}\gamma_{j+1})}{\gamma_{j+1}-\gamma_j}\right)\displaystyle\sum_{m=0}^{\infty}\dfrac{\overline{w}_U^{\,m+1}(\gamma_{j+1})}{\gamma_{j+1}^{\,2}}A_i\end{array}\right]$$

The second moment of the random variable T is derived by differentiating twice the Laplace transform of T with respect to s and at s = 0. From these results the variance of time to recruitment for the present model is determined.

Model-II:

In Model-II, inter-policy decision times are assumed to form a sequence of exchangeable and constantly correlated exponential random variables with mean λ and correlation $R\in[-1,1]$ with relation $b=\lambda(1-R)$. Taking derivative for the Laplace transform of the random variable T with respect to s and at s = 0, the mean time to recruitment for the present case is derived. In Eq.(3), we substitute

$$T_D=\left[\begin{array}{l}\left[\dfrac{\psi_1\psi_2-(1-F)f_1}{\psi_1\psi_2}\right]\displaystyle\sum_{m=0}^{\infty}\left(\dfrac{-mb}{(1-R)}-\dfrac{(m+1)b}{(1-R)}\right)E^m\\-\left[\dfrac{(1-F)(\psi_1f_2-f_1)}{(\psi_2-\psi_1)\psi_1^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\dfrac{(1-R)(1+b\psi_1)^{1-m}}{(1-R)+b\psi_1(1-R+mR)}\right)E^m\\+\left[\dfrac{(1-F)(\psi_1f_2-f_1)}{(\psi_2-\psi_1)\psi_1^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\dfrac{(1-R)(1+b\psi_1)^{-m}}{(1-R)+b\psi_1(1+mR)}\right)E^m\\+\left[\dfrac{(1-F)(f_1-f_2\psi_2)}{(\psi_2-\psi_1)\psi_2^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\dfrac{(1-R)(1+b\psi_2)^{1-m}}{(1-R)+b\psi_2(1-R+mR)}\right)E^m\\+\left[\dfrac{(1-F)(f_1-f_2\psi_2)}{(\psi_2-\psi_1)\psi_2^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\dfrac{(1-R)(1+b\psi_2)^{-m}}{(1-R)+b\psi_2(1+mR)}\right)E^m\end{array}\right]$$

For i=1,2,3 and j=1,3,5

$$T_i=\left[\begin{array}{l}\left[\dfrac{\gamma_j\gamma_{j+1}-(1-B_i)d_j}{\gamma_j\gamma_{j+1}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\dfrac{-mb}{(1-R)}-\dfrac{(m+1)b}{(1-R)}\right)A_i^m\\-\left[\dfrac{(1-B_i)(\gamma_jd_{j+1}-d_j)}{(\gamma_{j+1}-\gamma_j)\gamma_j^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\dfrac{(1-R)(1+b\gamma_j)^{1-m}}{(1-R)+b\gamma_j(1-R+mR)}\right)A_i^m\\+\left[\dfrac{(1-B_i)(\gamma_jd_{j+1}-d_j)}{(\gamma_{j+1}-\gamma_j)\gamma_j^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\dfrac{(1-R)(1+b\gamma_j)^{-m}}{(1-R)+b\gamma_j(1+mR)}\right)A_i^m\\+\left[\dfrac{(1-B_i)(d_j-d_{j+1}\gamma_{j+1})}{(\gamma_{j+1}-\gamma_j)\gamma_{j+1}^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\dfrac{(1-R)(1+b\gamma_{j+1})^{1-m}}{(1-R)+b\gamma_{j+1}(1-R+mR)}\right)A_i^m\\+\left[\dfrac{(1-B_i)(d_j-d_{j+1}\gamma_{j+1})}{(\gamma_{j+1}-\gamma_j)\gamma_{j+1}^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\dfrac{(1-R)(1+b\gamma_{j+1})^{-m}}{(1-R)+b\gamma_{j+1}(1+mR)}\right)A_i^m\end{array}\right]$$

The second moment of the random variable T is derived by differentiating twice the Laplace transform of T with respect to s and at s = 0. From these results the variance of time to recruitment for the present model is determined.

Model-II:

In Model-III, inter-policy decision times are assumed to form a geometric process with independent and identically distributed hyper-exponential distribution representing the time between (i-1)th and ith policy decisions with mean $(p_1/\lambda_1)+((1-p_1)/\lambda_2)$, 0< $p_1<1$,where p_1 is the proportion of policy decisions having high attrition rate $\lambda_1>0$ and $\left(1-p_1\right)$ is the proportion of policy decisions having low attrition rate $\lambda_2>0$. Taking derivative for the Laplace transform of the random variable T with respect to s and at s = 0, the mean time to recruitment for the present case is derived. In Eq.(3), we substitute

$$T_D = \left[\begin{array}{l} \left[\dfrac{\psi_1\psi_2-(1-F)f_1}{\psi_1\psi_2}\right]\displaystyle\sum_{m=0}^{\infty}\left(\begin{array}{l}\left\{-\left[\dfrac{p_1}{\lambda_1}+\dfrac{(1-p_1)}{\lambda_2}\right]\left[\dfrac{a^m-1}{a^{m-1}(a-1)}\right]\right\}\\ -\left\{-\left[\dfrac{p_1}{\lambda_1}+\dfrac{(1-p_1)}{\lambda_2}\right]\left[\dfrac{a^{m+1}-1}{a^m(a-1)}\right]\right\}\end{array}\right)E^m \\ -\left[\dfrac{(1-F)(\psi_1 f_2-f_1)}{(\psi_2-\psi_1)\psi_1^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\prod_{r=1}^{m}\overline{w}_U{}^{(r)}\left(\dfrac{\psi_1}{a^{r-1}}\right)\right)E^m \\ +\left[\dfrac{(1-F)(\psi_1 f_2-f_1)}{(\psi_2-\psi_1)\psi_1^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\prod_{r=1}^{m+1}\overline{w}_U{}^{(r)}\left(\dfrac{\psi_1}{a^{r-1}}\right)\right)E^m \\ -\left[\dfrac{(1-F)(f_1-f_2\psi_2)}{(\psi_2-\psi_1)\psi_2^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\prod_{r=1}^{m}\overline{w}_U{}^{(r)}\left(\dfrac{\psi_2}{a^{r-1}}\right)\right)E^m \\ +\left[\dfrac{(1-F)(f_1-f_2\psi_2)}{(\psi_2-\psi_1)\psi_2^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\prod_{r=1}^{m+1}\overline{w}_U{}^{(r)}\left(\dfrac{\psi_2}{a^{r-1}}\right)\right)E^m \end{array}\right]$$

For i=1,2,3 and j=1,3,5

$$T_i = \left[\begin{array}{l} \left[\dfrac{\gamma_j\gamma_{j+1}-(1-B_i)d_j}{\gamma_j\gamma_{j+1}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\begin{array}{l}\left\{-\left[\dfrac{p_1}{\lambda_1}+\dfrac{(1-p_1)}{\lambda_2}\right]\left[\dfrac{a^m-1}{a^{m-1}(a-1)}\right]\right\}\\ -\left\{-\left[\dfrac{p_1}{\lambda_1}+\dfrac{(1-p_1)}{\lambda_2}\right]\left[\dfrac{a^{m+1}-1}{a^m(a-1)}\right]\right\}\end{array}\right)A_i^m \\ -\left[\dfrac{(1-B_i)(\gamma_j d_{j+1}-d_j)}{(\gamma_{j+1}-\gamma_j)\gamma_j^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\prod_{r=1}^{m}\overline{w}_U{}^{(r)}\left(\dfrac{\gamma_j}{a^{r-1}}\right)\right)A_i^m \\ +\left[\dfrac{(1-B_i)(\gamma_j d_{j+1}-d_j)}{(\gamma_{j+1}-\gamma_j)\gamma_j^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\prod_{r=1}^{m+1}\overline{w}_U{}^{(r)}\left(\dfrac{\gamma_j}{a^{r-1}}\right)\right)A_i^m \\ +\left[\dfrac{(1-B_i)(d_j-d_{j+1}\gamma_{j+1})}{(\gamma_{j+1}-\gamma_j)\gamma_{j+1}^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\prod_{r=1}^{m}\overline{w}_U{}^{(r)}\left(\dfrac{\gamma_{j+1}}{a^{r-1}}\right)\right)A_i^m \\ +\left[\dfrac{(1-B_i)(d_j-d_{j+1}\gamma_{j+1})}{(\gamma_{j+1}-\gamma_j)\gamma_{j+1}^{\,2}}\right]\displaystyle\sum_{m=0}^{\infty}\left(\prod_{r=1}^{m+1}\overline{w}_U{}^{(r)}\left(\dfrac{\gamma_{j+1}}{a^{r-1}}\right)\right)A_i^m \end{array}\right]$$

The second moment of the random variable T is derived by differentiating twice the Laplace transform of T with respect to s and at s = 0. From these results the variance of time to recruitment for the present model is determined.

In all the above cases, the following notations are used

$$C_1 = \frac{\theta_B\theta_C\theta_D(\theta_B-\theta_C)}{(\theta_A-\theta_B)(\theta_B-\theta_C)(\theta_A-\theta_C)(\theta_A-\theta_D)};$$

$$C_2 = \frac{\theta_A\theta_C\theta_D(\theta_A-\theta_C)}{(\theta_A-\theta_B)(\theta_B-\theta_C)(\theta_A-\theta_C)(\theta_B-\theta_D)};$$

$$C_3 = \frac{\theta_A\theta_B\theta_D(\theta_A-\theta_B)}{(\theta_A-\theta_B)(\theta_B-\theta_C)(\theta_A-\theta_C)(\theta_C-\theta_D)};$$

$$E = \overline{w}_{X_A}(\theta_D)\overline{w}_{X_B}(\theta_D)\overline{w}_{X_C}(\theta_D); F = \overline{w}_{Y_A}(\theta_D)\overline{w}_{Y_B}(\theta_D)\overline{w}_{Y_C}(\theta_D);$$

$$A_1 = \overline{w}_{X_A}(\theta_A)\overline{w}_{X_B}(\theta_A)\overline{w}_{X_C}(\theta_A); B_1 = \overline{w}_{Y_A}(\theta_A)\overline{w}_{Y_B}(\theta_A)\overline{w}_{Y_C}(\theta_A);$$

$$A_2 = \overline{w}_{X_A}(\theta_B)\overline{w}_{X_B}(\theta_B)\overline{w}_{X_C}(\theta_B); B_2 = \overline{w}_{Y_A}(\theta_B)\overline{w}_{Y_B}(\theta_B)\overline{w}_{Y_C}(\theta_B);$$

$$A_3 = \overline{w}_{X_A}(\theta_C)\overline{w}_{X_B}(\theta_C)\overline{w}_{X_C}(\theta_C); B_3 = \overline{w}_{Y_A}(\theta_C)\overline{w}_{Y_B}(\theta_C)\overline{w}_{Y_C}(\theta_C);$$

Here, $\overline{w}_{X_A}(s)\overline{w}_{X_B}(s)\overline{w}_{X_C}(s) = \left[\dfrac{(1-R_1)(1+b_1 s)^{1-m}}{(1-R_1)+b_1 s(1-R_1+mR_1)}\right]$

$$\left[\frac{(1-R_2)(1+b_2 s)^{1-m}}{(1-R_2)+b_2 s(1-R_2+mR_2)}\right]\left[\frac{(1-R_3)(1+b_3 s)^{1-m}}{(1-R_3)+b_3 s(1-R_3+mR_3)}\right];$$

$$f_1 = d_1 = d_3 = d_5 = p_2\lambda_3\lambda_4 + (1-p_2)\lambda_3\lambda_4;$$

$$f_2 = d_2 = d_4 = d_6 = p_2\lambda_3 + (1-p_2)\lambda_4;$$

$$f_1^{\,*} = \lambda_3 + \lambda_4 - Fp_2\lambda_3 - F(1-p_2)\lambda_4;$$

$$f_2^{\,*} = \lambda_3\lambda_4 - Fp_2\lambda_3\lambda_4 - F(1-p_2)\lambda_3\lambda_4;$$

$$\psi_1 = \frac{f_1^{\,*} - \sqrt{f_1^{\,*2} - 4f_2^{\,*}}}{2};\quad \psi_2 = \frac{f_1^{\,*} + \sqrt{f_1^{\,*2} - 4f_2^{\,*}}}{2};$$

For i=1,2,3 and j=1,3,5

$$d_j^{\,*} = \lambda_3 + \lambda_4 - B_i p_2\lambda_3 - B_i(1-p_2)\lambda_4;$$

$$d_{j+1}^{\,*} = \lambda_3\lambda_4 - B_i p_2\lambda_3\lambda_4 - B_i(1-p_2)\lambda_3\lambda_4;$$

$$\gamma_j = \frac{d_j^{\,*} - \sqrt{d_j^{\,*2} - 4d_{j+1}^{\,*}}}{2};\quad \gamma_{j+1} = \frac{d_j^{\,*} + \sqrt{d_j^{\,*2} - 4d_{j+1}^{\,*}}}{2};$$

$$\overline{w}_U(s) = \frac{p_1\lambda_1}{\lambda_1+s} + \frac{(1-p_1)\lambda_2}{\lambda_2+s}; \overline{w}_U(0) = 1;$$

$$\overline{w}_U{}'(s) = -\frac{p_1\lambda_1}{(\lambda_1+s)^2} - \frac{(1-p_1)\lambda_2}{(\lambda_2+s)^2}; \overline{w}_U{}'(0) = -\frac{p_1}{\lambda_1} - \frac{(1-p_1)}{\lambda_2};$$

$$\overline{w}_U{}''(s) = \frac{2p_1\lambda_1}{(\lambda_1+s)^3} + \frac{(1-p_1)\lambda_2}{(\lambda_2+s)^3}; \overline{w}_U{}''(0) = \frac{2p_1}{\lambda_1^{\,2}} + \frac{2(1-p_1)}{\lambda_2^{\,2}};$$

$$E \neq F \neq A_i \neq B_i \neq 1, i = 1,2,3$$

$$\psi_1 \neq \psi_2, \gamma_j \neq \gamma_{j+1}, j = 1,3,5$$

2.2. *Numerical Illustration*

The mean and variance of time to recruitment for all the models are numerically illustrated by using OCTAVE.

Model-I:

The effect of the nodal parameters R_1, R_2 and R_3 on the mean and variance of time to recruitment are shown in the following Table 1.

Table 1. ($\alpha_{1A} = 0.5; \alpha_{1B} = 0.6; \alpha_{1C} = 0.7; \alpha_{2A} = 0.5; \alpha_{2B} = 0.6; \alpha_{2C} = 0.7; p_1 = 0.4; p_2 = 0.5; \theta_A = 0.4; \theta_B = 0.48; \theta_C = 0.52; \theta_D = 0.5; \lambda_1 = 0.8; \lambda_2 = 0.6; \lambda_3 = 0.6; \lambda_4 = 0.8$)

R_1	R_2	R_3	Model-I	
			E(T)	V(T)
-0.4	0.3	0.4	1.4389	6.5831
-0.3	0.3	0.4	1.4403	6.6071
0.8	0.3	0.4	1.4290	6.4138
0.9	0.3	0.4	1.4255	6.3519
0.3	-0.2	0.4	1.4335	6.6331
0.3	-0.1	0.4	1.4313	6.6485
0.3	0.5	0.4	1.4383	6.5231
0.3	0.6	0.4	1.4370	6.4643
0.3	0.5	-0.9	1.4341	6.1771
0.3	0.5	-0.8	1.4352	6.2709
0.3	0.5	0.5	1.4378	6.4772
0.3	0.5	0.6	1.4371	6.4203

Model-II:

The effect of the nodal parameters R_1, R_2, R_3 and R on the mean and variance of time to recruitment are shown in the following Table 2.

Table 2. ($\alpha_{1A} = 0.5; \alpha_{1B} = 0.79; \alpha_{1C} = 0.8; \alpha_{2A} = 0.85; \alpha_{2B} = 0.91; \alpha_{2C} = 0.9; p_2 = 0.2; \theta_A = 0.8; \theta_B = 0.78; \theta_C = 0.85; \theta_D = 0.86; \lambda = 0.75; \lambda_3 = 0.8; \lambda_4 = 0.89$)

R_1	R_2	R_3	R	Model-II	
				E(T)	V(T)
-0.7	0.91	0.2	0.3	4.0044	5.3353
-0.6	0.91	0.2	0.3	4.0070	5.3179
0.3	0.91	0.2	0.3	4.0123	5.2826
0.4	0.91	0.2	0.3	4.0110	5.2916
0.91	-0.9	0.2	0.3	3.9979	5.3757
0.91	-0.8	0.2	0.3	4.0036	5.3189
0.91	0.2	0.2	0.3	4.0217	5.1053
0.91	0.3	0.2	0.3	4.0203	5.1237
0.91	0.2	-0.9	0.3	4.0404	5.0525
0.91	0.2	-0.8	0.3	4.0385	5.0505
0.91	0.2	0.1	0.3	4.0205	5.1103
0.91	0.2	0.2	0.3	4.0217	5.1053
0.3	0.91	0.2	-0.11	4.4947	0.9756
0.3	0.91	0.2	-0.10	4.4677	1.2175
0.3	0.91	0.2	0.1	4.0672	4.6844
0.3	0.91	0.2	0.2	3.9866	5.3977

Model-III:

The effect of the nodal parameter a on the mean and variance of time to recruitment are shown in the following.

Table 3. ($\alpha_{1A} = 0.5; \alpha_{1B} = 0.6; \alpha_{1C} = 0.7; \alpha_{2A} = 0.4; \alpha_{2B} = 0.5; \alpha_{2C} = 0.6; p_1 = 0.5; p_2 = 0.6; \theta_A = 0.7; \theta_B = 0.8; \theta_C = 0.9; \theta_D = 0.6; \lambda_1 = 0.4; \lambda_2 = 0.3; \lambda_3 = 0.5; \lambda_4 = 0.6$)

R_1	R_2	R_3	a	Model-III	
				E(T)	V(T)
0.6	0.7	0.8	2	2.6261	5.6564
0.6	0.7	0.8	3	3.3330	10.8240
0.6	0.7	0.8	0.8	2.2619	1.3517
0.6	0.7	0.8	0.9	2.1749	1.2116

3. CONCLUSION

In Table 1,

1. For the increasing negative values of $\boldsymbol{R_1}, \boldsymbol{R_2}$ and $\boldsymbol{R_3}$, the mean increase for $\boldsymbol{R_1}, \boldsymbol{R_3}$ and decrease for $\boldsymbol{R_2}$ and the variance of time to recruitment increase.
2. For the increasing positive values of $\boldsymbol{R_1}, \boldsymbol{R_2}$ and $\boldsymbol{R_3}$, the mean decrease and the variance of time to recruitment decrease.

In Table 2,

1. For the increasing negative values of $\boldsymbol{R_1}$, the mean increase and the variance of time to recruitment decrease and for the increasing positive values of $\boldsymbol{R_1}$, the mean decrease and the variance of time to recruitment increase.
2. For the increasing negative values of $\boldsymbol{R_2}$, the mean increase and the variance of time to recruitment decrease and for the increasing positive values of $\boldsymbol{R_2}$, the mean decrease and the variance of time to recruitment increase.
3. For the increasing negative values of $\boldsymbol{R_3}$, the mean and the variance of time to recruitment decrease and for the increasing positive values of $\boldsymbol{R_3}$, the mean increase and the variance of time to recruitment decrease.
4. For the increasing negative and positive values of $\boldsymbol{R}$, the mean decrease and the variance of time to recruitment increase.

In Table 3,

1. When $\boldsymbol{0 < a < 1}$ and $\boldsymbol{a}$ increases, the mean and the variance of time to recruitment decrease for $\boldsymbol{a}$.
2. When $\boldsymbol{a > 1}$ and $\boldsymbol{a}$ increases, the mean and the variance of time to recruitment increase for $\boldsymbol{a}$.

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